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Computers and Mathematics with Applications 56 (2008) 590–591

An International Journal computers & mathematics with applications

www.elsevier.com/locate/camwa

Erratum

Erratum to: "Oscillation of forced neutral differential equations with positive and negative coefficients" [Comput. Math. Appl. 54 (2007) 1411–1421]

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Received 27 November 2007; accepted 30 November 2007

In [1], the author studied the oscillation of forced neutral delay differential equations with positive and negative coefficients, having the following form:

$$[x(t) - R(t)x(t-r)]' + P(t)x(t-\tau) - Q(t)x(t-\sigma) = f(t), \tag{1}$$

where $R, P, Q \in \mathcal{C}([t_0, \infty), \mathbb{R}^+)$ with $r > 0$, $\tau \geq \sigma \geq 0$. The readers are referred to [1] for further details.

The author uses an ill representation which yields a mistake. To avoid the mistake, we introduce the following representations:

$$Z(t,k) := R(t) + \int_{t-k}^{t} Q(u)\mathrm{d}u + \int_{t}^{t-k+\tau-\sigma} P(u)\mathrm{d}u$$
$$H(t,k) := P(t-k+\tau-\sigma) - Q(t-k),$$

and

$$z(t,k) := x(t) - R(t)x(t-r) - \int_{t-k}^{t} Q(u)x(u-\sigma)\mathrm{d}u - \int_{t}^{t-k+\tau-\sigma} P(u)x(u-\tau)\mathrm{d}u - \int_{t}^{\infty} f(u)\mathrm{d}u \tag{2}$$

for $k \in D := [0, \tau-\sigma]$ and $t \geq t_0 + \tau - \sigma$. By the representation $z'(t,k)$, we mean its derivative with respect to the first component t.

For the rest of the paper, we suppose that $H(t,k) \geq 0 (\not\equiv 0)$ eventually for $k \in D$. Now we restate [1, Lemma 1, Lemma 2 and Theorem 1] with the new representation to reveal the hidden mistake.

Lemma 0.1 *([1, Lemma 1]). Assume that there exists a real number $k_0 \in D$ such that $Z(t,k_0) \leq 1$ for $t \geq t_0$. Then,*

(i) *if x is an eventually positive solution of (1), then $z'(t,k_0) \leq 0$ and $z(t,k_0) > 0$ eventually.*
(ii) *if x is an eventually negative solution of (1), then $z'(t,k_0) \geq 0$ and $z(t,k_0) < 0$ eventually.*

DOI of original article: 10.1016/j.camwa.2007.03.010.
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doi:10.1016/j.camwa.2007.11.049

Let the constant $\rho(k)$ be defined as in [1, Section 1].

Lemma 0.2 ([1, Lemma 2]). *Assume that*

$$\int_t^\infty f(u)\mathrm{d}u \quad \text{is nondecreasing with} \quad \lim_{t\to\infty}\int_t^\infty f(u)\mathrm{d}u = 0 \tag{3}$$

and there exists a real number $k_0 \in D$ *such that* $Z(t,k_0) \geq 1$ *for* $t \geq t_0$. *Further assume that the second-order differential inequality*

$$y''(t) + \frac{1}{\rho(k_0)}H(t,k_0)y(t) \leq 0, \quad \text{for } t \geq t_0 \tag{4}$$

has no eventually positive solutions. Then,

(i) *if* x *is an eventually positive solution of* (1), *then* $z'(t,k_0) \leq 0$ *and* $z(t,k_0) < 0$ *eventually.*
(ii) *if* x *is an eventually negative solution of* (1), *then* $z'(t,k_0) \geq 0$ *and* $z(t,k_0) > 0$ *eventually.*

Now we state and discuss the ill theorem.

Theorem 0.3 ([1, Theorem 1]). *Assume that* (3) *holds and there exist two real numbers* $k_0, k_1 \in D$ *such that* $Z(t,k_0) \leq 1$ *while* $Z(t,k_1) \geq 1$ *eventually. Further assume that*

$$y''(t) + \frac{1}{\rho(k_1)}H(t,k_1)y(t) \leq 0, \quad \text{for } t \geq t_0$$

has no eventually positive solutions, then every solution of (1) *oscillates.*

In the proof of [1, Theorem 1], it is considered that $z(t,k_0) = z(t,k_1)$. This is the mistake which is hidden by the ill representation in the proofs of [1].

Now we state the correction of [1, Theorem 2.1] below.

Theorem 0.4 (*Correction of [1, Theorem 1]*). *Assume that* (3) *holds and there exists* $k_0 \in D$ *satisfying* $Z(t,k_0) \equiv 1$ *eventually and* (4) *has no eventually positive solutions, then every solution of* (1) *is oscillatory.*

Proof. In the present case, if x is an eventually positive (negative) solution of (1), then $z(t,k_0)$ is eventually positive by Lemma 0.1, while it is eventually negative (positive) by Lemma 0.2. This is a contradiction. Therefore, (1) has no eventually positive solutions. □

The same problem appears in the rest of the paper [1], which can be corrected in a similar way to that in Theorem 0.4.

References

[1] Özkan Öcalan, Oscillation of forced neutral differential equations with positive and negative coefficients, Comput. Math. Appl. 54 (2007) 1411–1421.